

1. (20%) A capacitor C is charged up to a potential V and connected to an inductor L . At time $t = 0$ the switch S is closed.
 - (a) Find the current in the circuit as a function of time. (10%)
 - (b) How does your answer change if a resistor R is included in series with C and L . (10%)

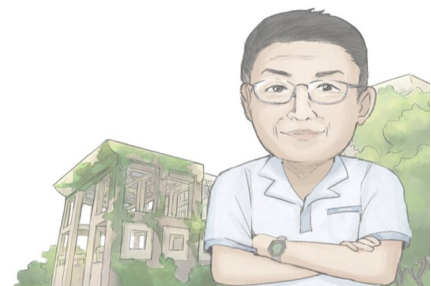
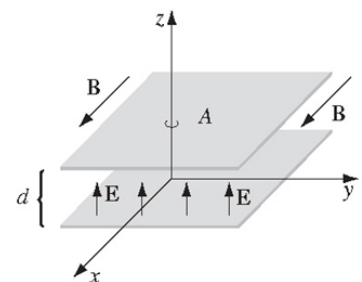
2. (20%) Derive the Poynting theorem in **matter**, starting from the power delivered per unit volume

$$\frac{dW}{dt} = \int_V (\mathbf{E} \cdot \mathbf{J}_f) d\tau.$$

- (a) Show that the Poynting vector becomes $\mathbf{S} = \mathbf{E} \times \mathbf{H}$. (10%)
- (b) Show that the energy density for linear media is $u_{\text{em}} = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$. (10%)

3. (20%) A charged parallel-plate with uniform electric field ($\mathbf{E} = E \hat{\mathbf{z}}$) is placed in a uniform magnetic field ($\mathbf{B} = B \hat{\mathbf{x}}$) as shown in the figure below.
 - (a) Find the electromagnetic momentum in the space between the plates. (10%)
 - (b) Now a resistive wire is connected between the plates, along the z axis, so that the capacitor slowly discharges. The wire will experience a magnetic force. What is the total impulse delivered to the system, during the discharge? (10%)

[Hint: $\mathbf{F} = \oint_S \vec{\mathbf{T}} \cdot d\mathbf{a} - \epsilon_0 \mu_0 \frac{d}{dt} \int_V \mathbf{S} d\tau$ and $T_{ij} \equiv \epsilon_0 (E_i E_j - \frac{1}{2} \delta_{ij} E^2) + \frac{1}{\mu_0} (B_i B_j - \frac{1}{2} \delta_{ij} B^2)$]



4. (25%) A hollow rectangular waveguide has a cross section of $a \times b = 2.54 \text{ mm} \times 1.27 \text{ mm}$.
- (a) Estimate the cutoff frequencies for the first three modes (TE_{10} , TE_{20} , and TE_{01}). (10%)
- (b) If the waveguide is 5.0 mm long and are closed at both ends (forming a resonant cavity), calculate the resonant frequency of the TE_{101} ($m = 1$, $n = 0$, and $l = 1$) mode. (10%)
- (c) Qualitatively plot the dispersion relation ($\omega - k_z$ diagram) of the dominant TE_{10} mode. (5%)
- [Hint: Detail how to calculate and express your answers in parts (a) and (b) in terms of GHz.]

5. (25%) A plane wave of frequency ω , traveling in the z direction with the electric field polarized in the x direction, is incident from vacuum (ϵ_0 and μ_0) to a dielectric with $\epsilon = 4\epsilon_0$ and $\mu = \mu_0$. The incident wave is

$$\tilde{\mathbf{E}}_I(z, t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \quad \text{and} \quad \tilde{\mathbf{B}}_I(z, t) = \frac{1}{c} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}}$$

where k is the corresponding wave number.

- (a) Write down the reflected wave ($\tilde{\mathbf{E}}_R(z, t)$, $\tilde{\mathbf{B}}_R(z, t)$) and the transmitted wave ($\tilde{\mathbf{E}}_T(z, t)$, $\tilde{\mathbf{B}}_T(z, t)$). (10%)
- (b) Express the reflected amplitude $\tilde{E}_{0R}$ and transmitted amplitude $\tilde{E}_{0T}$ in terms of the incident amplitude ($\tilde{E}_{0I}$). (10%)
- (c) Find the reflection coefficient ($R \equiv I_R/I_I$) and the transmission coefficient ($T \equiv I_T/I_I$). (5%)
- [Hint: $I \equiv \langle S \rangle = \frac{1}{2} v \epsilon E_0^2$.]

